# SECTION E

# DERIVATION OF EQUATIONS FOR THERMAL EFFECTS ON WIRE ROPE CABLE INTERMEDIATE RAILINGS

## E.1—THERMAL EXPANSION AND CONTRACTION

### Symbols and Notations

$\alpha$ Thermal expansion coefficient for wire rope cable, in/in/°F.

$\alpha_c$ Thermal expansion coefficient for concrete, in/in/°F.

$\alpha_{eff}$ Effective thermal expansion coefficient considering effects of cables, frame and support, in/in/°F.

$\alpha_{frame}$ Average thermal expansion coefficient of the frame and supporting base, in/in/°F.

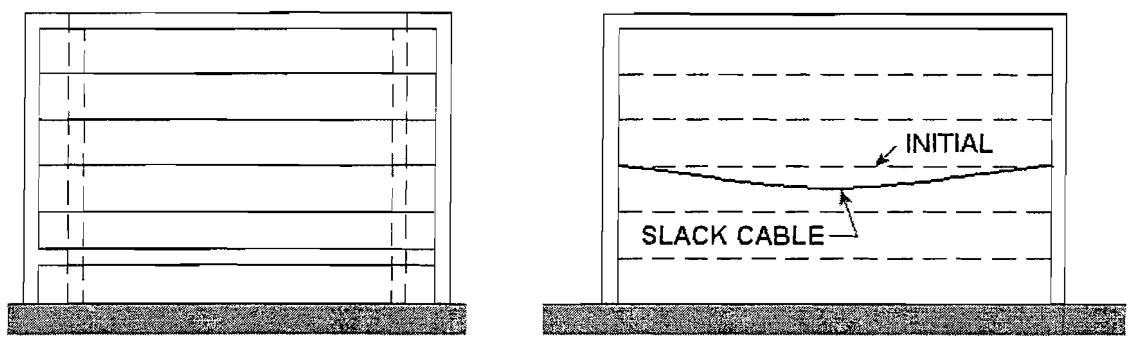
$\alpha_{stl}$ Thermal expansion coefficient for carbon steel, in/in/°F.

### Background

When an object changes temperature, it expands or contracts an amount proportional to the temperature change. Every material responds to a temperature change at a unique rate, known as the material's *thermal expansion coefficient*. The thermal expansion coefficient, $\alpha$, for 316 stainless steel wire rope is:

$$\alpha := 9.6 \cdot 10^{-6} \cdot \frac{\text{in}}{\text{in} \cdot {}^\circ\text{F}}$$

As temperature increases, the handrail and infill cables expand. This thermal expansion has opposite effects in the frame and the cables—as the top rail (and bottom rail of the frame, if so equipped) expands, it tends to make the cables more taut *(below, left)*, but as the cables themselves expand, they tend to go slack *(below, right)*.




**FRAME EXPANSION** **CABLE EXPANSION**

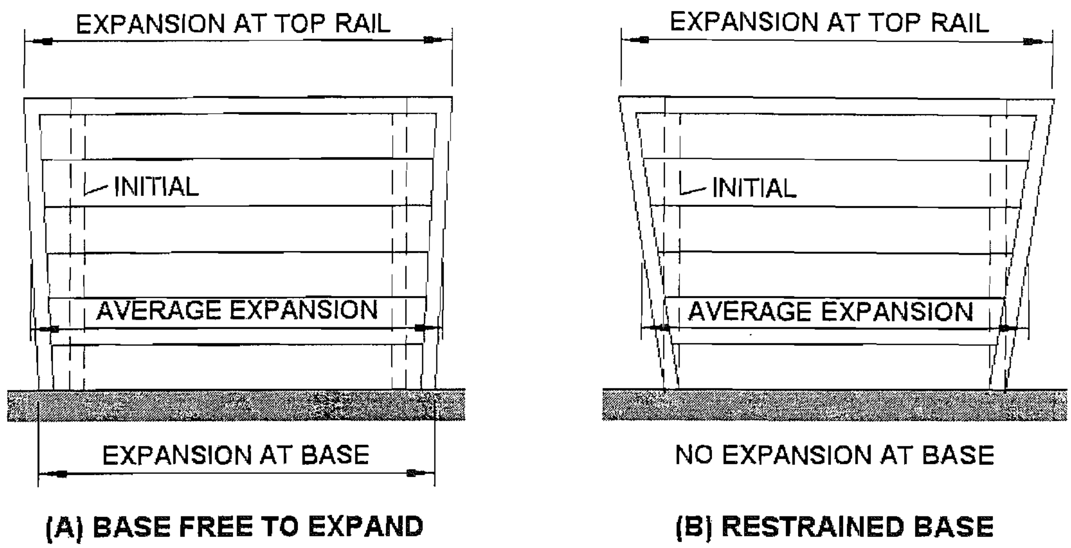
If the frame and the cables are of the same material, with the same thermal expansion coefficient, $\alpha$, and the same modulus of elasticity, *E*, the thermal strains in the the frame and in the cables would be equal, but of opposite signs, and no net change in cable tension would occur. This condition would be met, for example, if the frame, as well as the intermediate cable railings, were stainless steel. Often, however, the frame is carbon steel, which has a slightly different thermal expansion coefficient:

$$\alpha_{stl} := 6.5 \cdot 10^{-6} \cdot \frac{\text{in}}{\text{in} \cdot {}^\circ\text{F}}$$

If the frame has no bottom rail, the bottom of the frame is then the concrete slab, which has yet a different thermal expansion coefficient:

$$\alpha_c := 5.0 \cdot 10^{-6} \cdot \frac{in}{in \cdot °F}$$

The intermediate cables would thus see a tensioning effect from the frame that varied between the thermal strain in the concrete and the thermal strain in the carbon steel top rail *(see below)*.




On the average, the frame would react as if the thermal expansion coefficient were the average of the coefficients for carbon steel and concrete. If the concrete were restrained from movement, as in a balcony enclosed on three sides, the average thermal expansion coefficient would simply be one-half of the coefficient value for carbon steel. This is the (conservative) case we will use in these equations:

$$\alpha_{frame} := 3.25 \cdot 10^{-6} \cdot \frac{in}{in \cdot °F}$$

Since the thermal expansion coefficient for the stainless steel wire rope infill cables is greater than the frame's value, the cables will tend to expand more than the frame, and the effective expansion will be the numerical difference between the two:

$$\alpha_{eff} := \alpha - \alpha_{frame}$$

$$\alpha_{eff} = 6.35 \times 10^{-6} \frac{in}{in \cdot °F}$$

For a stainless steel frame, the average thermal expansion coefficient would simply be one-half of the coefficient value for stainless steel:

$$\alpha_{SSframe} := 4.8 \cdot 10^{-6} \cdot \frac{in}{in \cdot °F}$$

Similarly, the effective thermal expansion coefficient would be:

$$\alpha_{SSeff} := \alpha - \alpha_{SSframe}$$

$$\alpha_{SSeff} = 4.8 \times 10^{-6} \frac{in}{in \cdot °F}$$

Because the effective thermal expansion coefficient for stainless steel frames is less than for carbon steel frames, the thermal effects on systems with stainless steel frames are less. Therefore, using $\alpha_{eff}$ for both materials provides conservative results, and will be used exclusively for the remainder of this discussion.

# E.2—EFFECT ON PRESTRESSING FORCE

### Symbols and Notations

$\alpha_{eff}$ Effective thermal expansion coefficient considering effects of cables, frame and support, in/in/°F.

$\Delta F_{psT}$ Change in prestress force due to temperature change, lbs.

$\Delta T$ Change in temperature, °F

$\varepsilon$ Strain, in/in.

$\sigma$ Stress, ksi.

$A$ Cross-sectional area, $in^2$.

$D$ Diameter of wire rope cable, in.

$E$ Modulus of Elasticity, ksi.

$E_{eff}$ Effective Modulus of Elasticity, ksi.

### Strain Change to Prestress Force Change

Thermal expansion or contraction can be quantified, since the resulting strain change is directly proportional to the temperature change:

$$\Delta\varepsilon = \alpha_{eff} \cdot \Delta T$$

Because our railing infill cables are prestressed (pretensioned) and thus restrained from changing length, the strain change in the cable manifests itself as an increase or decrease in the stress in the cable. This change in stress is given by:

$$\Delta\sigma = \Delta\varepsilon \cdot E$$

The change in prestress force is the product of the stress change and the area of the cable.

$$\Delta F_{psT} = -\Delta\sigma \cdot A$$

$$\Delta F_{psT} = -\alpha_{eff} \cdot \Delta T \cdot E \cdot A$$

**Mathcad Function:**

$$\Delta F_{psT}(\Delta T, D) := \left| \begin{array}{l} A \leftarrow \dfrac{\pi \cdot D^2}{4} \\ -\alpha_{eff} \cdot \Delta T \cdot E_{eff} \cdot A \end{array} \right.$$

**Example**: Given a ten degree temperature change, compute the change in prestress force for a 3/8" wire rope:

$$\Delta T := 10 \cdot °F$$

$$D := 0.375 \cdot in$$

$$\Delta F_{psT}(\Delta T, D) = -114.3\,lbf$$

# E.3—EFFECTS OF TEMPERATURE INCREASE ON SPHERE PASS-THROUGH RESISTANCE

### Symbols and Notations

| | |
|---|---|
| $\Delta$ | Deflection of cable under mid-span point load, *P*. |
| $\Delta F_{psT}$ | Change in prestress force due to temperature change, lbs. |
| $\Delta T$ | Change in temperature, °F. |
| $D$ | Diameter of wire rope cable, in. |
| $F_{ps}$ | Applied prestressing force, lbs. |
| $L$ | Spacing between intermediate supports, in. |
| $L_T$ | Length of cable between anchor points, ft. |
| $P$ | Mid-span point load required to produce deflection Δ, lbs. |
| $P_b$ | Component of mid-span point load, *P*, resisted by flexural bending, lbs. |
| $P_{ef}$ | Component of mid-span point load, *P*, resisted by stretching of cable, lbs. |
| $P_{ps}$ | Component of mid-span point load, *P*, resisted by cable prestressing, lbs. |

### Temperature Effects

As derived previously, changes in temperature and the resulting expansion and contraction in the wire-rope cable, can be incorporated as a change in the effective prestress force:

$$\Delta F_{psT} = -\alpha_{eff} \cdot \Delta T \cdot E \cdot A$$

Therefore, the effective prestress force, $F'_{ps}$ becomes:

$$F'_{ps} = F_{ps} + \Delta F_{psT}$$

### Mathcad Functions, including Temperature Effects

Incorporating temperature effects,

$$F'_{ps} := F_{ps} + \Delta F_{psT}(\Delta T, D)$$

the Mathcad function for the load-deflection relationship that was derived in Section A can be rewritten:

$$P(\Delta, D, L, L_T, F_{ps}, \Delta T) := \left| \begin{array}{l} F'_{ps} \leftarrow F_{ps} + \Delta F_{psT}(\Delta T, D) \\ P_{ef}(\Delta, D, L, L_T) + P_b(\Delta, D, L) + P_{ps}(F'_{ps}, \Delta, L) \end{array} \right.$$

Similarly, the equation for equilibrium deflection, Δeq, derived in Section A, can also be rewritten to include temperature effects:

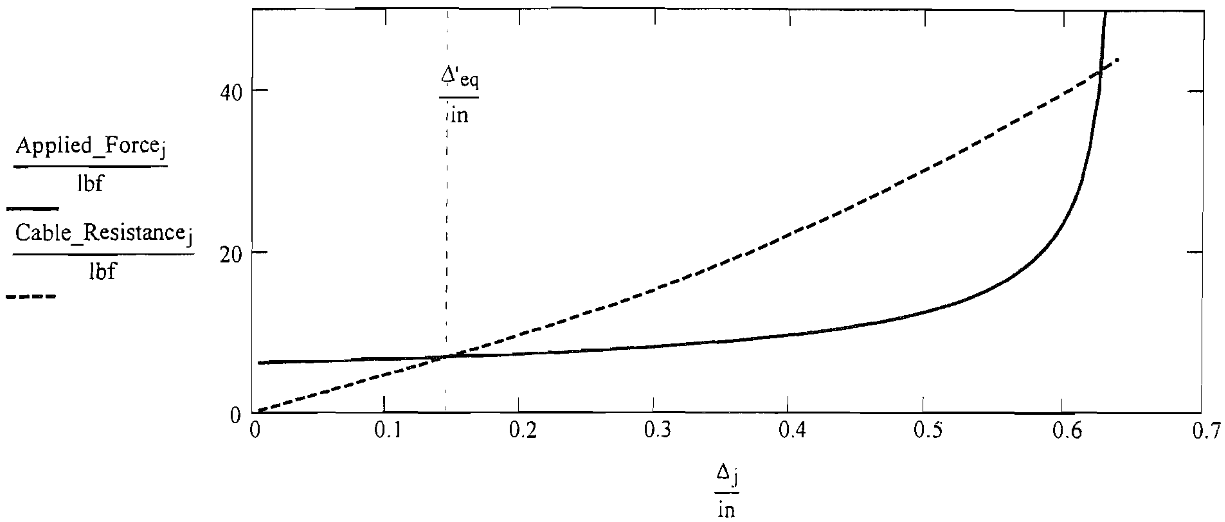
$$\Delta_{eq}(F_x, D_b, D, S_o, \Delta_{max}, L, L_T, F_{ps}, \Delta T) := \left| \begin{array}{l} \Delta \leftarrow 0 \cdot in \\ \Delta_{max} \text{ on error root}\left(\begin{array}{l} F(F_x, \Delta, D, D_b, S_o) \ldots \\ + -P(\Delta, D, L, L_T, F_{ps}, \Delta T) \end{array}, \Delta\right) \end{array} \right.$$

**Example 1:** Given the following parameters,

| | | | |
|---|---|---|---|
| Diameter of Cable: | $D := 0.375 \cdot in$ | Cable Spacing: | $S_o := 3.1 \cdot in$ |
| Support Spacing: | $L := 42 \cdot in$ | Anchor Spacing: | $L_T := 12 \cdot ft$ |
| Prestress Force: | $F_{ps} := 400 \cdot lbf$ | Temperature Change: | $\Delta T := 0 \cdot °F$ |
| Sphere Diameter: | $D_b := 4 \cdot in$ | Load on Sphere | $F_x := 8.7 \cdot lbf$ |

Determine the equilibrium deflection, and plot the load vs. deflection curves.

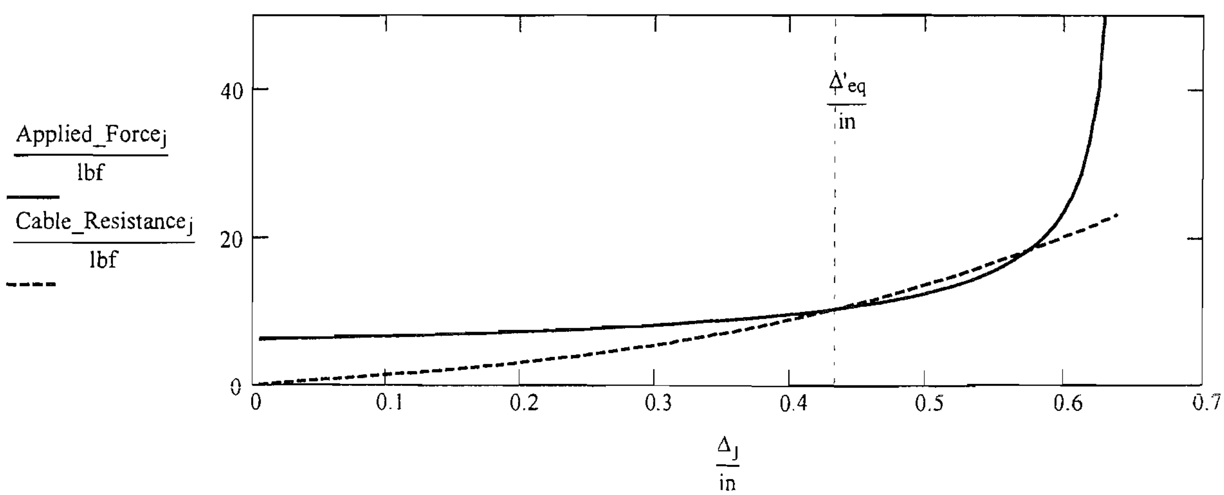
Equilibrium Deflection: $\Delta_{eq}\left(F_x, D_b, D, S_o, \Delta_{max}(D, D_b, S_o), L, L_T, F_{ps}, \Delta T\right) = 0.147\,in$




**Example 2:** Given the same parameters, but with a 30°F increase in temperature, determine the equilibrium deflection, and plot the load vs. deflection curves.

Temperature Change: $\Delta T := 30 \cdot °F$

Equilibrium Deflection: $\Delta_{eq}\left(F_x, D_b, D, S_o, \Delta_{max}(D, D_b, S_o), L, L_T, F_{ps}, \Delta T\right) = 0.434\,in$

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